RESISTANCE PREDICTION OF SMALL HIGH-SPEED DISPLACEMENT VESSELS: STATE OF THE ART*

by

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Summary

In preliminary ship design studies it is frequently necessary to estimate the calm water resistance characteristics of various hull forms prior to carrying out model tests. For such estimations, use is generally made of results of well-known methodical model experiments such as Taylor's Standard Series, Series 60, and others, to determine the effect of specific hull form parameters on resistance. For small, high-speed displacement vessels designed to operate in the speed range corresponding to Froude number values of 0.4 to 1.1 (equivalent to a range in $V/\sqrt{L}$ of 1.34 to 3.70), these well-known methodical series results are inadequate due to the limited speed range covered. Alternative methods must then be used. In this paper, all available and reliable data for the prediction of the resistance of small, high-speed vessels, designed to operate in the displacement mode, are presented. Included are the results of restricted and less well-known methodical series and averaged results of tests with a large number of non-systematic models, in both graphical and numerical form. Some basic considerations on how and when each of the presented methods can be applied are also presented.

1. Introduction

Methods for the prediction of the hydrodynamic resistance of ships are used in preliminary ship design studies when the influence of displacement, length and hull form on speed and power have to be determined. Model tests are usually only carried out once these first design considerations have resulted in a more-or-less definite design. The aim of this paper is to compile all useful data on the resistance of high-speed, round bilge displacement vessels to facilitate preliminary design studies on the effect of hull dimensions and hull form on ship speed and required power for this kind of vessel.

For most ships approximate resistance predictions can be carried out by means of methods based on well-known methodical series experiments such as Taylor's Standard Series, Series 60, etc. These methods present the results of resistance tests with models constituting a systematic series whereby it is possible to identify the effect on resistance of various hull form parameters. The hull form parameters which are usually adopted in an extensive series are the length-displacement ratio $L/\nabla^{1/3}$ (or $\Delta/(0.01L)^3$) or the length-breadth ratio L/B, the breadth-draught ratio B/T, and either the block coefficient C_B or the prismatic coefficient C_P. Most methodical series experiments cover a speed range which is not sufficient for application to high-speed displacement vessels, at speeds in excess of a Froude number value of 0.4 (equivalent to $V/\sqrt{L}$ = 1.34). Some less extensive methodical series experiments, however, are available for this purpose, in addition to a number of numerical methods. These methods are presented in the main part of this paper. The presentation of the original results of some of the graphical methods have been re-arranged for the sake of wanting to obtain one particular presentation throughout this paper, a presentation which facilitates application of the results in a particularly straight forward manner.

2. Basic considerations

The type of ship addressed in this paper is that which is designed to operate at speeds at which fully developed planing is impossible. Fully developed planing is possible at speeds in excess of a Froude number value of about 1.1 (equivalent to $V/\sqrt{L}$ = 3.70), if a hard-chine type of hull form with flat underwater sections is adopted. At speeds below this value of the Froude number no hydrodynamic advantages in adopting a hard-chine type of hull exist. Both the resistance and the seakeeping behaviour of a round-bilge type of hull are then generally superior. For this reason, ships which operate in a displacement mode, below the above-mentioned speed value, are designed as round-bilge hulls. At speeds in excess of the speed at which the primary resistance hump occurs (at a Froude number value of about 0.5), these hull forms experience some lift due to the action of dynamic forces which increase as speed is increased. The significant decrease in wetted surface due to the bow being lifted clear of the water, such as occurs with hard-chine forms, does not occur, however.

In the context of this paper, the term high speed is meant to indicate speeds in excess of a Froude number value of about 0.4 (equivalent to $V/\sqrt{L}$ = 1.34). This

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Table 1.
Ship length and ship speed values leading to Froude number values of 0.4 and 1.1.

Ship length in metres	Ship length in feet	Ship speed in knots for $F_n = 0.4 (V/\sqrt{L} = 1.34)$	Ship speed in knots for $F_n = 1.1 (V/L = 3.70)$
5	16.40	5.45	14.98
10	32.81	7.70	21.19
20	65.62	10.90	29.97
40	131.23	15.41	42.38
60	196.85	18.87	51.91
80	262.47	21.79	59.94
100	328.08	24.36	67.02
200	656.17	34.33	94.78

definition of high speed finds its origin in the fact that the wave-making resistance becomes significant at this Froude number value, requiring a more slender hull form in order to keep the power of the required prime mover within acceptable limits. Some combinations of length and speed, resulting in the above-mentioned values of the Froude number, are given in Table 1.

Unfortunately, in presenting the results of model resistance tests, different formats are used by different authorities. No normalized procedure exists because of specific advantages of certain presentations in certain cases. When presenting the results of tests of a systematic model series, for general use, it is advantageous to adopt a format in which residual resistance values are used rather than total resistance values. To justify this statement it is necessary to consider the fact that the resistance of a ship is composed of viscous and non-viscous components. The frictional resistance R_F is dependent on the Reynolds number while the non-viscous or residual resistance R_R, mainly consisting of wave resistance, is dependent on the Froude number, i.e.

$$R_T = R_F(R_n) + R_R(F_n) \tag{1}$$

It follows that if the total resistance were to be adopted in presenting the results of a systematic model series for general use, it would be necessary to incorporate the Reynolds number and the Froude number as independent variables (or separate speed and length or Froude number and length variables). This constitutes a particular handicap when presenting the results graphically.

In equation (1), the frictional resistance is assumed to be dependent on the wetted surface S, the square of the ship speed V, the mass density of water ρ and the coefficient of friction C_F as follows:

$$R_F = \tfrac{1}{2} \rho\, V^2\, S\, C_F \tag{2}$$

In this equation, C_F is dependent on the value of the Reynolds number R_n. This formula is used to estimate the frictional resistance for the model (to determine the residual resistance from the measured total resistance) and for the ship (to determine the total resistance from the residual resistance). The residual resistance of the model, expressed e.g. as a fraction of the displacement weight, is equal to the residual resistance of the ship when model tests are carried out at full-scale values of the Froude number (rather than at full-scale values of the Reynolds number).

During the last years, alternative formulas have been developed for the frictional resistance R_F. These are based on the knowledge that the frictional resistance is influenced by the three-dimensional form of the vessel. In assuming that the frictional resistance of a ship is equal to that of a flat plate with an equivalent area of wetted surface, the frictional resistance is underestimated leading to an overestimation of the residual resistance. Considerable uncertainty about the value of this form effect exists, however. This practise has not been incorporated in this paper, primarily because this would entail a new analysis of the results of methodical model series which do not incorporate the influence of this form effect. In some cases a re-analysis is not possible because of lack of information concerning the measured values of the total resistance of the models (in some cases these values have not been published). On using the same friction formulation as was originally used in arriving at the published residual resistance values, however, and on using a realisitic value of the model-ship correlation factor C_A, no serious errors need occur on using a two-dimensional frictional resistance formulation. The actual procedure for calculating the resistance and effective power of a proposed design, to be used in the context of the results given in the main part of this paper, is as follows. The total resistance is calculated from the following equation:

$$R_T = \tfrac{1}{2} \rho\, S V^2 (C_F + C_A) + \frac{R_R}{\Delta}\, \Delta \tag{3}$$

in which

relation of this parameter with other commonly-used length-displacement (or displacement-length) ratios is as follows:

$$L/\nabla^{1/3} = \frac{30.57}{(\Delta/(0.01L)^3)^{1/3}} \tag{8}$$

where, in $\Delta/(0.01L)^3$, Δ is the displacement in tons of 2240 lbs and L the waterline length in feet.

$$L/\nabla^{1/3} = \frac{10}{(\nabla/(0.1L)^3)^{1/3}} \tag{9}$$

where $\nabla/(0.1L)^3$ is non-dimensional.

The speed parameter used in this paper in presenting residual resistance values of methodical model series is the volumetric Froude number $F_{n\nabla}$, defined as:

$$F_{n\nabla} = \frac{V}{\sqrt{g\nabla^{1/3}}} \tag{10}$$

where V is the ship speed in m/sec., g the acceleration due to gravity (= 9.81 m/sec.^{2}) and ∇ the volume of displacement in m^3. This speed parameter allows a better comparison of residual resistance values for ships of different length-displacement ratio, such as occur in methodical model series. The relation of this parameter with other commonly-used speed parameters is as follows:

$$F_{n\nabla} = \sqrt{\frac{L}{\nabla^{1/3}}} \cdot F_n \quad \text{where } F_n = \frac{V}{\sqrt{gL}} \tag{11}$$

and

$$F_{n\nabla} = 0.2975 \sqrt{\frac{L}{\nabla^{1/3}}} \cdot \frac{V}{\sqrt{L}} \tag{12}$$

where, in $V/\sqrt{L}$, V is the ship speed in knots and L is the waterline length in feet. All other dimensions of V, L, g and ∇ in equations (10) and (11) are in metric (SI) units.

The wetted surface of systematic hull forms is often expressed as $S = C_S\nabla^{2/3}$ or $S = C_S\sqrt{\nabla L}$ where C_S is a constant for a particular hull form. In this paper, $S = C_S\sqrt{\nabla L}$ is used. To facilitate adopting the alternative expression, use can be made of the following relation.

$$\frac{S}{\nabla^{2/3}} = \sqrt{\frac{L}{\nabla^{1/3}}} \cdot \frac{S}{\sqrt{\nabla L}} \tag{13}$$

3. Resistance prediction by methodical series data

3.1. Nordstrom Series

In 1936, Nordstrom [1]* published the results of tests carried out at the Royal Institute of Technology in Stockholm with 14 different round bilge models, 5 of which were tested at more than 1 draught. Three of these models, each tested at 3 different draughts, form a small systematic series.

The results of resistance tests with these models, carried out in calm water, were originally analyzed with Froude's frictional resistance coefficients. Even though no turbulence stimulating devices were adopted, subsequent analyses of the results have not revealed low enough resistance values to suspect significant influence of laminar flow.

The results obtained by Nordstrom are now only rarely used, with the exception of the results for the small systematic series. As originally presented, the results are only useful for a full-scale displacement range of between 10 and 30 m^3. A more applicable presentation of these results is given in Figure 1, in which the residual resistance-displacement weight ratio R_R/Δ is given as a function of the length-displacement ratio $L/\nabla^{1/3}$ and the volumetric Froude number $F_{n\nabla}$. The residual resistance values in Figure 1 are based on a re-analysis carried out by De Groot [2] using the 1947 ATTC friction line with $C_A = 0$.

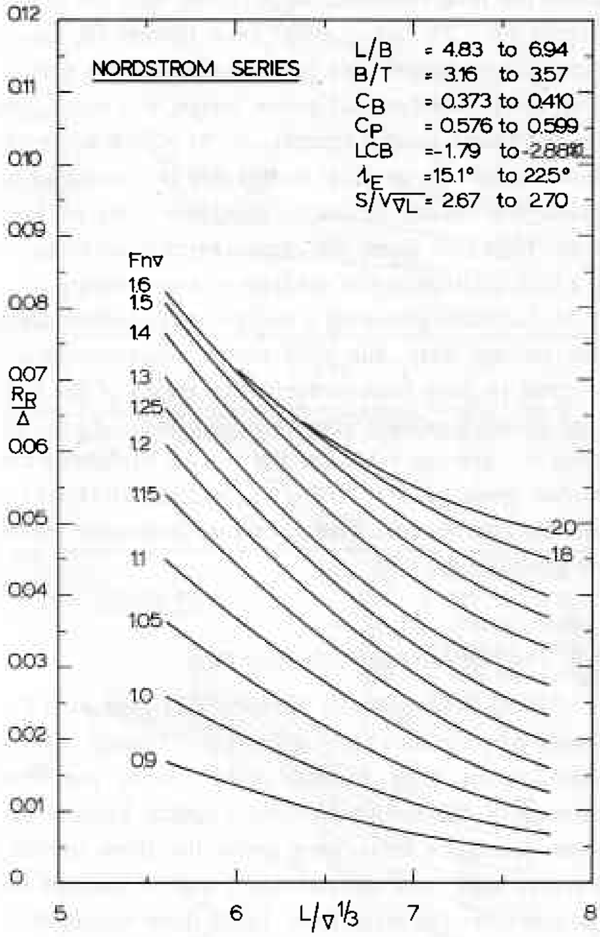

Figure 1. Averaged value of the residual resistance-displacement weight ratio R_R/Δ of the Nordstrom methodical series.

3.2. De Groot Series

In 1951, De Groot [2] published the results of tests with 31 round bilge, high-speed hull forms, 7 of which

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*) Numbers in brackets designate references listed at the end of the paper.

were tested at 2 or more draughts. These models were tested at the Netherlands Ship Model Basin (NSMB). Four of these 31 models, each tested at 4 draughts, constitute a small systematic series. Tests with these 4 models were also carried out in the towing tank of the Delft University of Technology.

The results of resistance tests with all 31 models, carried out in calm water, were analysed using the 1947 ATTC friction coefficients. For the small systematic series, De Groot presented a diagram displaying the values of the residual resistance coefficient $C_R = R_R/½\rho SV^2$ as a function of $V/\sqrt{L}$ and the displacement-length ratio $\nabla/(0.1L)^3$ which diagram is often referred to by other workers in the field. More benefit of the work carried out by De Groot, however, lies in the fact that he derived a single graph (Figure 8 of Reference [2]), showing the average resistance of 76 models (including the Nordstrom data) as a function of the displacement-length ratio for the speed range corresponding to $V/\sqrt{L} = 1.0$ to 3.5. Unfortunately, the total resistance values given were for model lengths of 2.25 metres only. Even though De Groot showed how to use this graph in arriving at a prediction of the resistance of a new design, this procedure has not found general acceptance. At NSMB however, these results are used up to this day as a standard to which the results of model resistance tests of fast, round-bilge hull forms are compared in order to arrive at a first qualification of the lines of a new design.

To facilitate preparing a resistance prediction using this average data, the total model resistance values referred to have been converted to values of the residual resistance-displacement weight ratio R_R/Δ by using the average value for the wetted surface of the models given by $S = 2.75\sqrt{\nabla L}$, and the 1947 ATTC friction coefficients. These residual resistance values are given in Figure 2.

3.3. The Marwood and Silverleaf Data

In 1960, Marwood and Silverleaf [3] presented the results of resistance tests with approximately 30 unrelated round-bilge forms carried out at the Ship Division of the British National Physical Laboratory. Mean resistance lines were given for these models, together with lines representing 5 and 10 percent deviations from the mean lines. From these results, Marwood and Silverleaf confirmed earlier observations by Nordstrom and De Groot that other than the length-displacement ratio, and the Froude number, no clear evidence of any important systematic variation in resistance with other parameters can be discerned in the speed range considered, contrary to the case for the low speed range and the planing speed range.

The Marwood and Silverleaf resistance data presents the resistance coefficient Ⓒ = EHP x $427.1/V^2\Delta^{2/3}$ for a range of values of the speed-length ratio and the displacement-length ratio for a standard length of 100 feet. The values of the speed-length ratio $V\sqrt{L}$ covered is from 1.4 to 3.5 and the values of the length-displacement ratio $L/\nabla^{1/3}$ covered is from 5.2 to 8.2. These results have not been converted to R_R/Δ-values here as was done for the De Groot data, since this would only form a duplication of the De Groot data, in that both present average results.

3.4. Series 63

Results of resistance tests with models of five 50 feet round-bilge utility boats were reported on by Beys [4] in 1963. These tests were carried out in the towing tank of the Davidson Laboratory of the Stevens Institute of Technology. These models form a methodical series in that the body plans of all 5 models are geometrically similar. The parent model has a nominal length-beam ratio of 4. All other models were derived from this parent model by multiplying the waterline and buttock spacings of the parent model by a con-

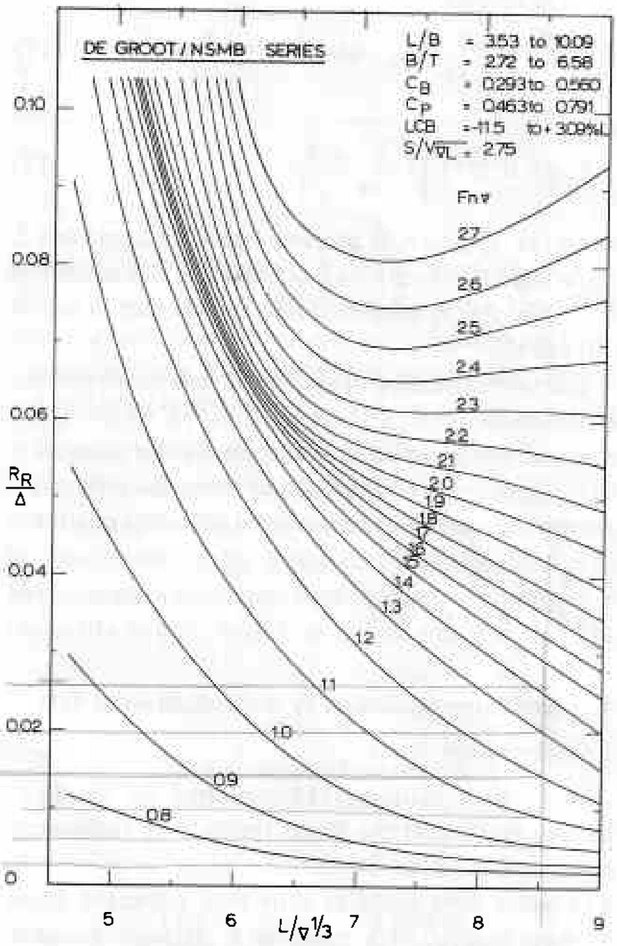

Figure 2. Residual resistance-displacement weight ratio R_R/Δ derived from the averaged, total resistance of 76 models compiled by De Groot.

stant so as to obtain nominal L/B values of 2.5, 3.0, 4.0, 5.0 and 6.0. The models had a length of 3 feet. Unfortunately, this procedure of deriving a systematic series leads to changes not only in L/B, but also in B/T (and hence in C_B, C_P, LCB, etc.) when tests at equal length-displacement ratios are carried out such as is the case here. To obtain the same $L/\nabla^{1/3}$ for the model with $L/B = 2.5$ as for the model with $L/B = 6.0$, for example, a relatively smaller draught has to be adopted for the model with the largest beam. Therefore, the differences in the resistance values between the different models at equal length-displacement and Froude number values cannot be attributed solely to differences in L/B only. Nevertheless, the results obtained with these 5 models are useful to designers requiring more information on the influence on resistance of hull form parameters other than the length-displacement ratio. The residual resistance-displacement weight ratio R_R/Δ for each of these models are, accordingly, given here in Figures 3, 4, 5, 6 and 7 in the format used in Figures 1 and 2. The presented residual resistance values were derived from the measured total resistance by means of the 1947 ATTC friction line with $C_A = 0$.

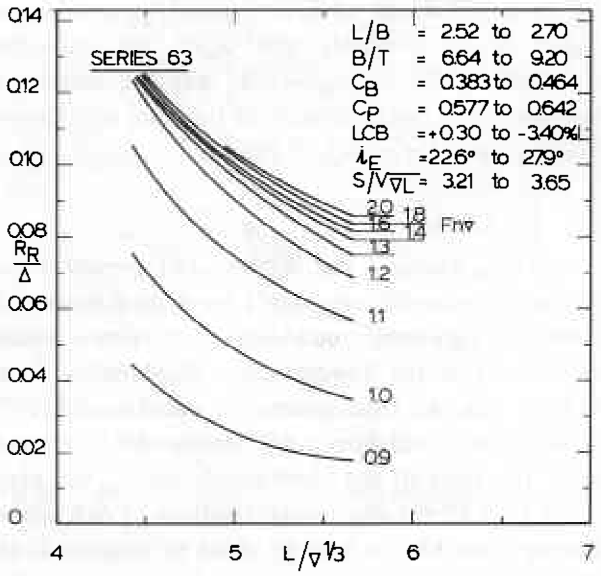

Figure 3. Residual resistance-displacement weight ratio R_R/Δ of the Series 63 methodical models with a nominal L/B value equal to 2.5.

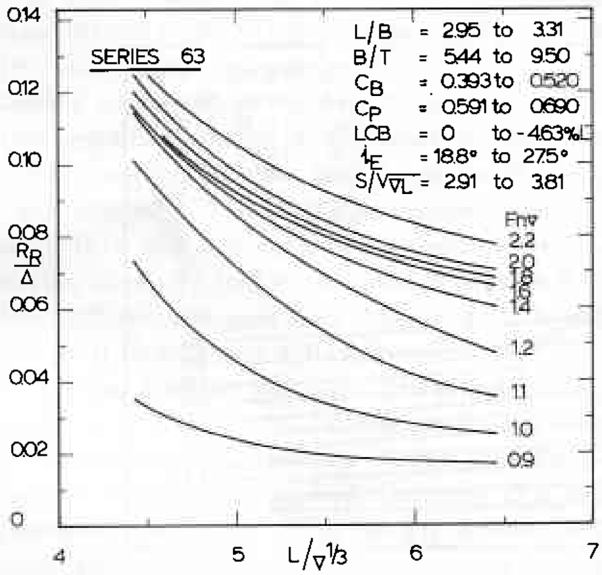

Figure 4. Residual resistance-displacement weight ratio R_R/Δ of the Series 63 methodical models with a nominal L/B value equal to 3.0.

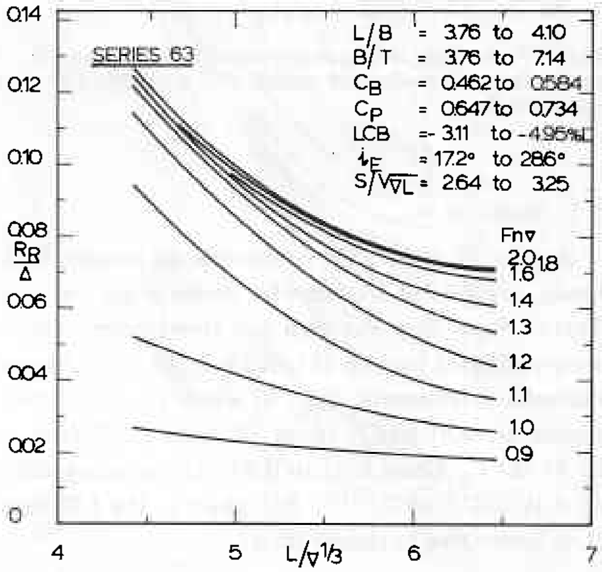

Figure 5. Residual resistance-displacement weight ratio R_R/Δ of the Series 63 methodical models with a nominal L/B value equal to 4.0.

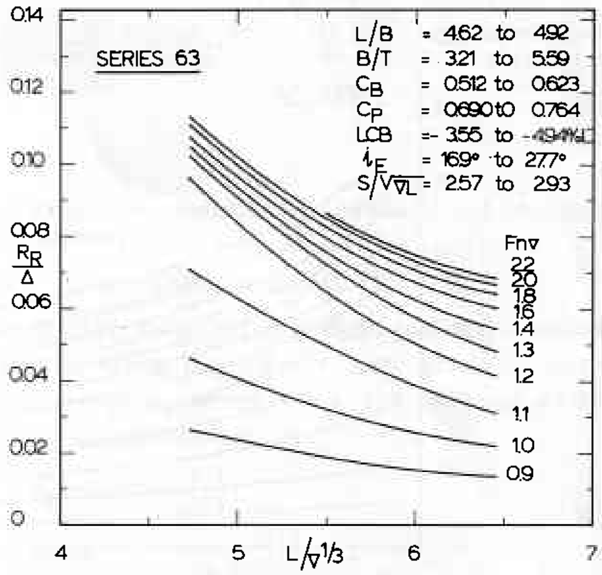

Figure 6. Residual resistance-displacement weight ratio R_R/Δ of the Series 63 methodical models with a nominal L/B value equal to 5.0.

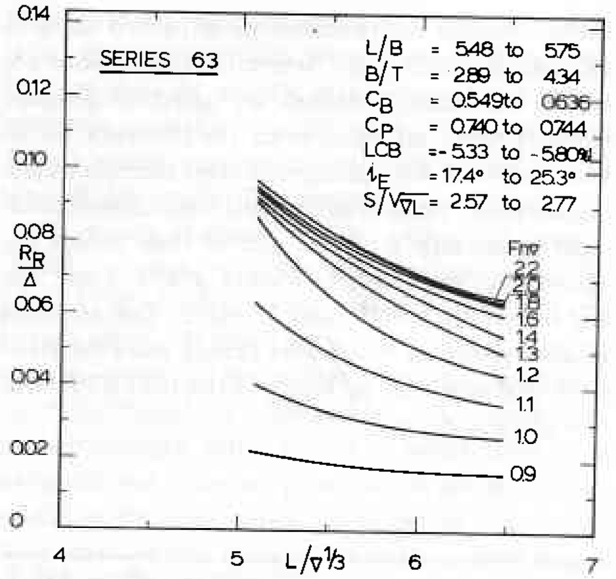

Figure 7. Residual resistance-displacement weight ratio R_R/Δ of the Series 63 methodical models with a nominal L/B value equal to 6.0.

3.5. *Series 64*

Results of tests with 27 models of slender, high-speed, round-bilge displacement forms at the David W. Taylor Naval Ship Research and Development Center were published by Yeh [5] in 1965. These 27 models comprise a systematic series of which the varied parameters are $\Delta/(0.01L)^3$ (from 15 to 55), B/T (from 2 to 4) and C_B (from 0.35 to 0.55). On assigning values to $\Delta/(0.01L)^3$ or $L/\nabla^{1/3}$, B/T and C_B, the L/B value is no longer free to choose since:

$$\frac{L}{B} = \sqrt{\frac{C_B(L/\nabla^{1/3})^3}{B/T}} \qquad (14)$$

The L/B values of these 27 models thus range from 8.454 to 18.264. The speed range covered corresponds to values of the Froude number F_n from 0 to 1.5 (equivalent to a range in $V/\sqrt{L}$ from 0 to 5.0). The results of the resistance tests in calm water were reduced to residuary resistance values by Yeh, by using the 1947 ATTC frictional resistance coefficients with $C_A = 0$.

Due to the rather extreme type of hull forms in this series, the resistance results for the individual models are not often used or referred to. Average resistance values for the complete series, however, are frequently adopted for use in parametric studies for slender ships and other purposes. For this reason the average residual resistance-displacement weight ratio R_R/Δ, as a function of $L/\nabla^{1/3}$ and $F_{n\nabla}$, is shown in Figure 8. In preparing this figure the data for $\Delta/(0.01L)^3 = 20$ and $C_B = 0.45$ was not considered because of the 'inconsistency' of this data with the results for the other models of the series.

3.6. *SSPA Series*

In 1968, Lindgren and Williams [6] presented the results of resistance tests with a methodical series of 9 models of high-speed, round-bilge displacement vessels carried out at the Swedish State Shipbuilding Tank (SSPA). The hull form parameters varied were $L/\nabla^{1/3}$ (values of 6, 7 and 8) and B/T (values of 3.0, 3.5 and 4.0). The value of the block coefficient C_B was kept equal to 0.40 for all models, resulting in L/B values ranging from 4.62 to 8.20 by virtue of equation (14). The speed range covered corresponds to a range in the Froude number F_n from 0.4 to 1.2 (equivalent to a range in $V/\sqrt{L}$ from 1.34 to 4.0).

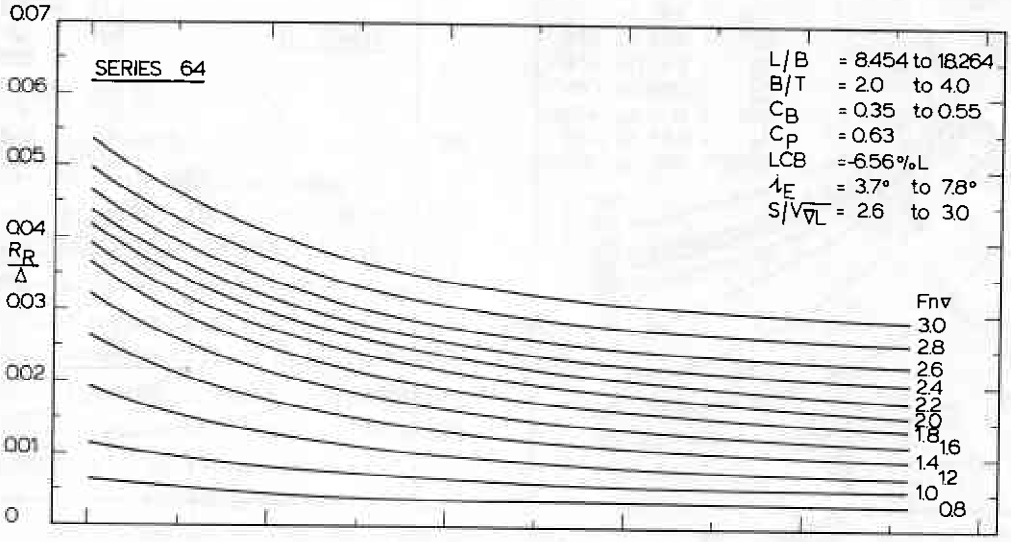

Figure 8. Averaged value of the residual resistance-displacement weight ratio R_R/Δ of the Series 64 methodical models.

The residuary resistance values of the models were obtained by using the 1957 ITTC frictional resistance coefficients. Up to a F_n-value equal to about 0.90, the results for the 3 B/T values are almost identical, again leading to the observation that in the speed range between F_n = 0.4 and about 0.9 the length-displacement ratio is the only significant parameter. For this speed range the residuary resistance-displacement weight ratio R_R/Δ of this series is shown in Figure 9, as a function of $L/\nabla^{1/3}$ and $F_{n\nabla}$.

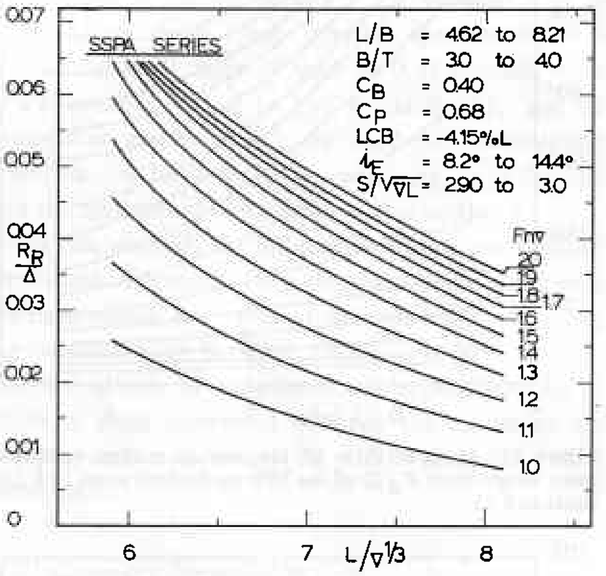

Figure 9. Averaged value of the residual resistance-displacement weight ratio R_R/Δ of the SSPA methodical series.

3.7. NPL Series

Very useful resistance data on high-speed, round bilge displacement forms have been published by Marwood and Bailey [7] in 1969 and by Bailey [8] in 1976, concerning the tests carried out at the Ship Division of the British National Physical Laboratory with a systematic series of 22 models of which L/B and B/T were varied. Five models were tested having a L/B value of 3.33 with B/T values ranging from 3.19 to 10.21, 6 with L/B equal to 4.54 with B/T values ranging from 1.72 to 6.87, 4 with L/B equal to 5.41 with B/T values ranging from 1.94 to 4.86, 4 with L/B equal to 6.25 with B/T values ranging from 1.93 to 5.80 and 3 models with $L/B = 7.50$ with B/T values ranging from 2.01 to 4.02. Other main hull form parameters were kept constant ($C_B = 0.397$, $C_P = 0.693$) and the longitudinal centre of buoyancy LCB was positioned 6.4% L aft of the midship section). The speed range covered corresponds to values of the Froude number ranging from 0.3 to 1.20 (equivalent to a range in $V/\sqrt{L}$ from 1.0 to 4.0).

The residuary resistance values were calculated from the measured model resistance by subtracting the frictional resistance as determined by means of the 1957 ITTC skin friction formulation. The residuary resistance-displacement weight ratio was then plotted against $L/\nabla^{1/3}$ for various $F_{n\nabla}$-values for each L/B value. These figures are reproduced here as Figures 10, 11, 12, 13 and 14.

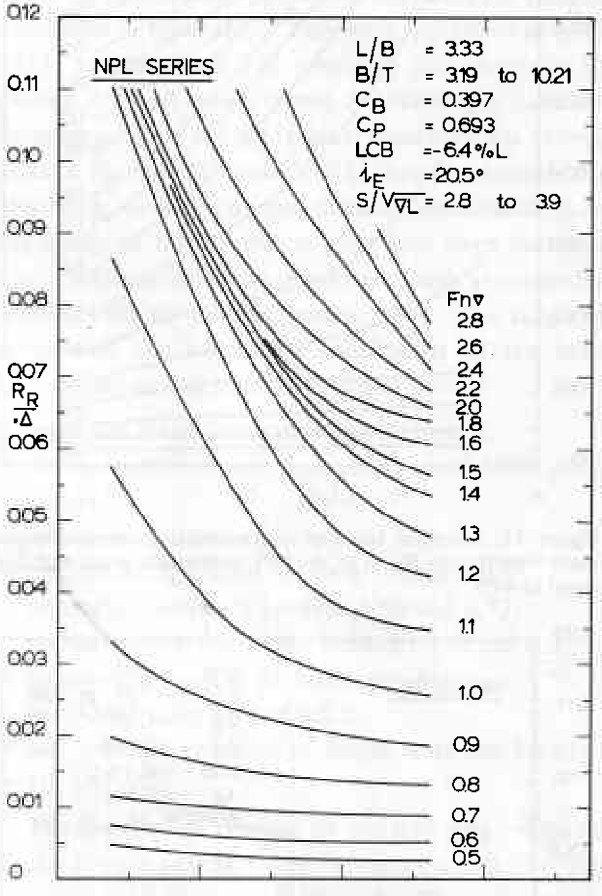

Figure 10. Averaged value of the residual resistance-displacement weight ratio R_R/Δ of the NPL methodical series with L/B equal to 3.33.

4. Resistance prediction by empirical and statistical methods

4.1. Kafali's Graphical Method

In 1959, Kafali [9] published a graphical procedure for estimating the effective horse power of small, round bilge high-speed vessels. The procedure adopts the following formula:

$$P_E = C\,\Delta\,V\left(\frac{V}{\sqrt{L}}\right)^2 \qquad (15)$$

in which

P_E = effective horse power,
Δ = displacement in tons,

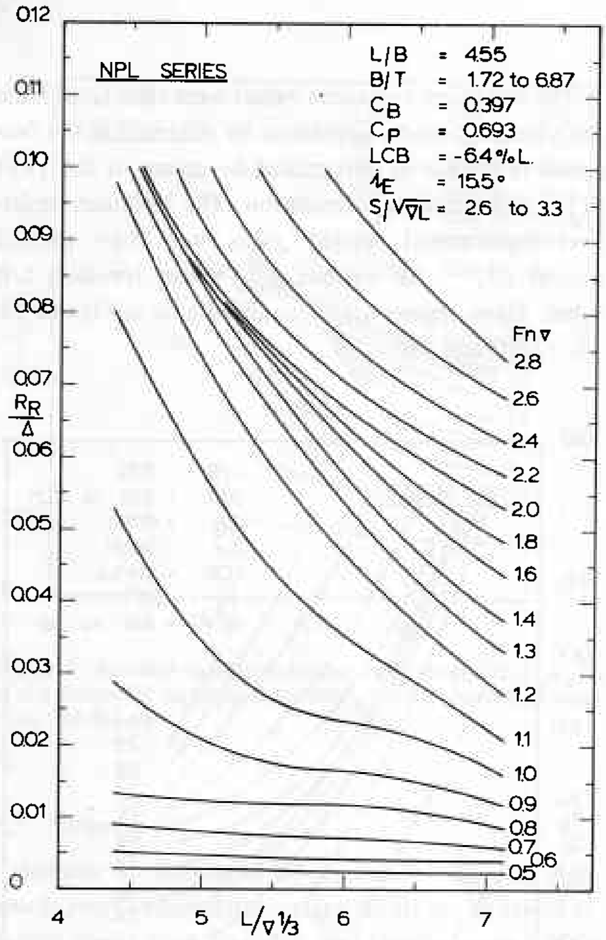

Figure 11. Averaged value of the residual resistance-displacement weight ratio R_R/Δ of the NPL methodical series with L/B equal to 4.55.

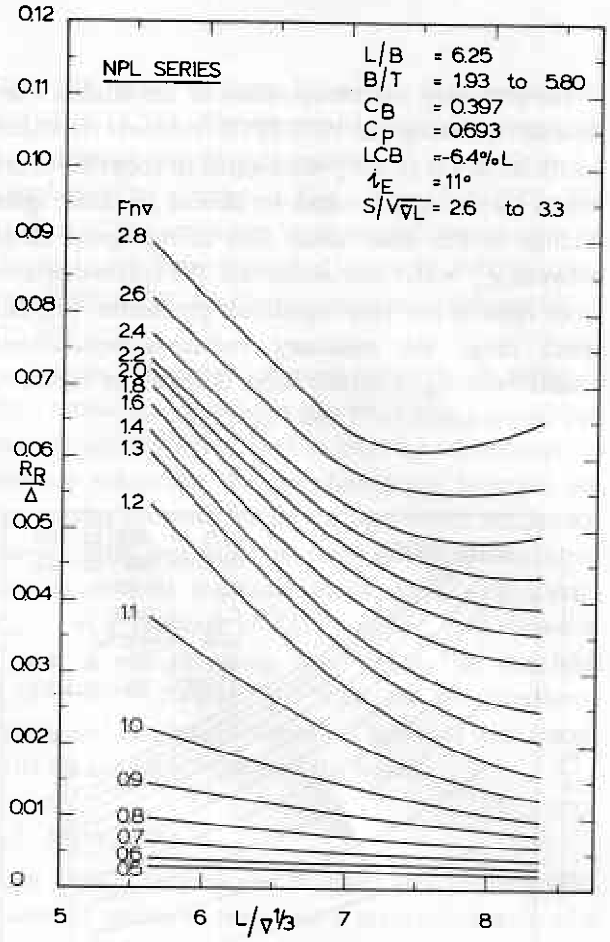

Figure 13. Averaged value of the residual resistance-displacement weight ratio R_R/Δ of the NPL methodical series with L/B equal to 6.25.

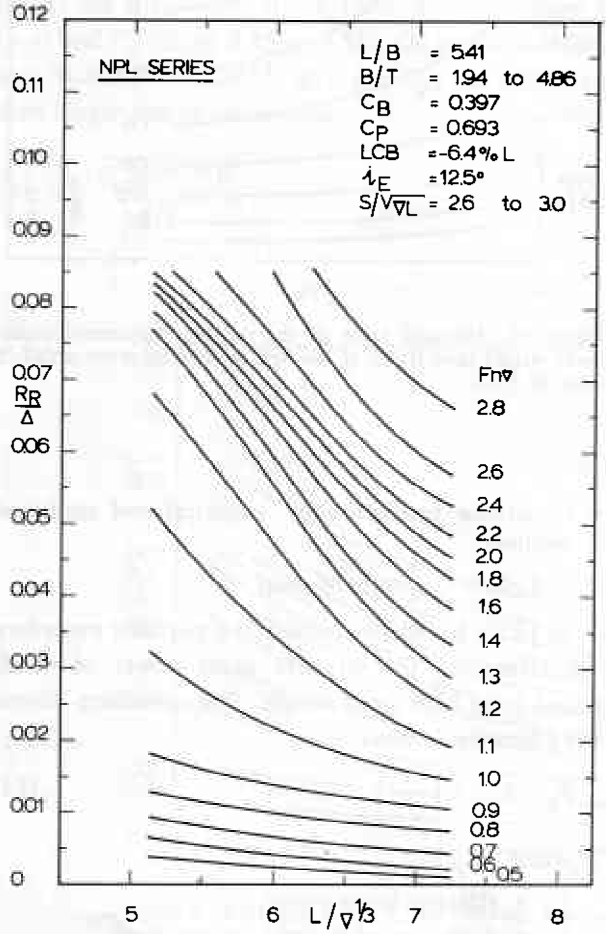

Figure 12. Averaged value of the residual resistance-displacement weight ratio R_R/Δ of the NPL methodical series with L/B equal to 5.41.

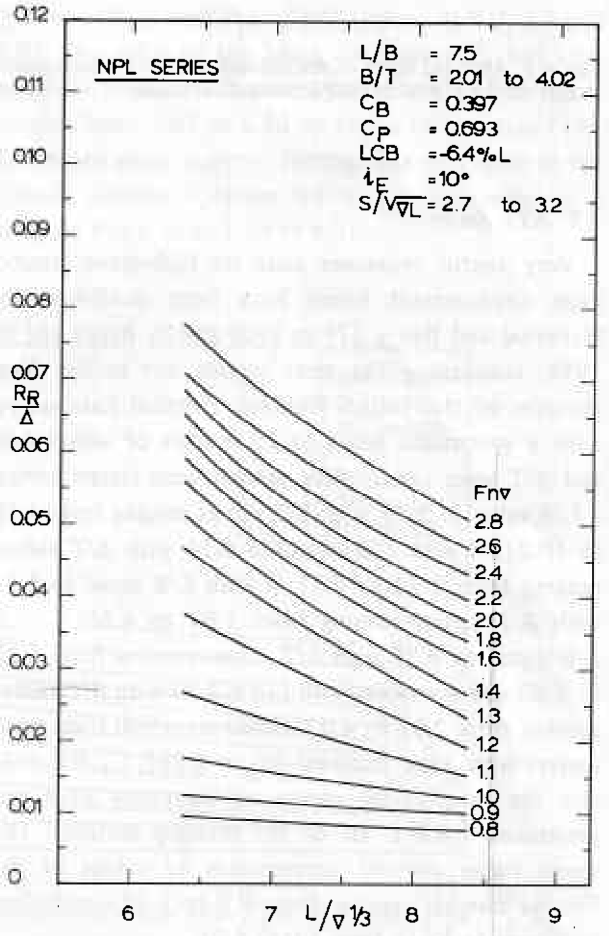

Figure 14. Averaged value of the residual resistance-displacement weight ratio R_R/Δ of the NPL methodical series with L/B equal to 7.5.

V = ship speed in knots,
L = waterline length in feet.

In equation (15), C is a constant, the value of which is to be determined from graphs. These graphs display C as a function of $V/\sqrt{L}$, $L/\Delta^{1/3}$ (where L is in feet and Δ is in tons) and B/T. One graph is provided for $B/T = 3.65$ and another for $B/T = 4.45$. In Reference [9], it is stated that the value of C was determined from model tests. No details of these tests are provided, however. The values for the length-displacement ratio adopted in these graphs range from 19 to 26 (equivalent to values of $L/\nabla^{1/3}$ of between 5.84 to 8.00). The speed range covered corresponds to a Froude number range of 0.35 to 0.85 (equivalent to $V/\sqrt{L}$ values from 1.2 to 2.9). Equation (15), and the associated graphs for C, are valid for the bare-hull case, i.e. for hulls without appendages, as is the case for the methodical series data given in section 3.

In this method, the influence of length on frictional resistance is ignored. Only one value for C_F was used in determining the value of the constant C. On using an allowance of between +5 and +10% to account for the effects of roughness and appendages, Kafali finds a good agreement between the results of the method and trial results for a 20.8 ft and a 112 ft motorboat,

4.2. Clement's Graphical Method

A graphical procedure for the prediction of the total resistance of small, round bilge, high-speed craft was published by Clement in 1964 [10]. The Nordstrom data [1] and the Marwood and Silverleaf data [3] were used by Clement to derive a set of eight graphs presenting the total resistance-weight ratio R_T/Δ, dependent on the displacement weight Δ, the length-displacement ratio $L/\nabla^{1/3}$ and the volumetric Froude number $F_{n\nabla}$. Each graph is valid for one value of $F_{n\nabla}$. Graphs are provided for $F_{n\nabla} = 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4$ and 2.6. The values of the length-displacement ratio covered by the graphs range from 5.2 to 8.2, while the displacement weight values range from 5000 to100000 pounds. As explained in section 2, a dependence on displacement or length is necessary when the total resistance is adopted as the dependent variable in lieu of the residual resistance because of the dependence of skin friction on the Reynolds number. In deriving the R_T/Δ-values given in the graphs, Clement made use of the Froude friction coefficients. He used values for the wetted surface following from the formula $S = 0.157L^2$ (with L in feet), which formula is valid for the average value of the (unrelated) models of Reference [3]. Application of the model-ship correlation factor C_A, as outlined in section 2, is not possible on using total resistance values derived from graphs such as presented by Clement. To arrive at realistic full-scale values of the resistance, use of an integral allowance of approximately +10% seems appropriate for small vessels [3].

4.3. Van Oortmerssen's Numerical Method

Van Oortmerssen used a multiple regression technique to obtain a numerical expression for the residual resistance of small ships. This work, published in 1971 [11], was based on 970 resistance data points of 93 models of small vessels tested at NSMB. The basic expression adopted for the residual resistance was derived from a theoretical model of the wave resistance of a travelling, two-dimensional pressure disturbance, having peaks at the equivalent stem and stern positions and a pressure minimum in between, such as occurs for vessels with no parallel middle body. The range of parameters for which the coefficients of the basic expressions are valid, are as follows:

- waterline length between 8 and 80 metres;
- displacement volume between 5 and 3000 cubic metres;
- length-beam ratio between 3 and 6.2;
- breadth-draught ratio between 1.9 and 4.0;
- prismatic coefficient between 0.50 and 0.73;
- midshipsection coefficient between 0.70 and 0.97;
- longitudinal centre of buoyancy between −7% L and +2.8% L forward of 0.5 L;
- half angle of entrance of design waterline between 10° and 46°.

The speed range covered by the 970 data points lie in the Froude number range between 0 and 0.50 (equivalent to a $V/\sqrt{L}$ range of 0 to 1.70). Some extrapolation to higher speeds is permissable, however, because of the theoretical nature of the basic expression. The numerical expression is as follows:

$$\frac{R_R}{\Delta} = C_1 e^{-mF_n^{-2}/9} + C_2 e^{-mF_n^{-2}} + C_3 e^{-mF_n^{-2}} \cdot \sin F_n^{-2} + + C_4 e^{-mF_n^{2}} \cdot \cos F_n^{-2} \qquad (16)$$

in which

$$\begin{aligned} 1000\,C_1 = {} & 79.32134 - 0.09287\,LCB + \\ & - 0.00209\,LCB^2 - 246.45896\,C_p + \\ & + 187.13664\,C_P^2 - 1.42893\,L/B + \\ & + 0.11898\,(L/B)^2 + 0.15727\,C_{WL} + \\ & - 0.00064\,C_{WL}^2 - 2.52862\,B/T + \\ & + 0.50619\,(B/T)^2 + 1.62851\,C_M \end{aligned}$$

$$\begin{aligned} 1000\,C_2 = {} & 6714.88397 + 19.83\,LCB + \\ & + 2.66997\,LCB^2 - 19662.024\,C_p + \\ & + 14099.904\,C_P^2 + 137.33613\,L/B + \end{aligned}$$

$$\begin{aligned} & -13.36938\,(L/B)^2 - 4.49852\,C_{WL} + \\ & + 0.021\,C_{WL}^2 + 216.44923\,B/T + \\ & - 35.07602\,(B/T)^2 - 128.72535\,C_M \end{aligned} \tag{17}$$

$$\begin{aligned} 1000\,C_3 = & -908.44371 + 2.52704\ LCB + \\ & - 0.35794\,LCB^2 + 755.1866\,C_P + \\ & - 48.93952\,C_P^2 + 9.86873\,L/B + \\ & - 0.77652\,(L/B)^2 + 3.79020\,C_{WL} + \\ & - 0.01879\,C_{WL}^2 - 9.24399\,B/T + \\ & + 1.28571\,(B/T)^2 + 250.6491\,C_M \end{aligned}$$

$$\begin{aligned} 1000\,C_4 = & \ 3012.14549 + 2.71437\,LCB + \\ & + 0.25521\,LCB^2 - 9198.8084\,C_P + \\ & + 6886.60416\,C_P^2 - 159.92694\,L/B + \\ & + 16.23621\,(L/B)^2 - 0.82014\,C_{WL} + \\ & + 0.00225\,C_{WL}^2 + 236.3797\,B/T + \\ & - 44.1782\,(B/T)^2 + 207.2558\,C_M \end{aligned}$$

and $\quad m = 0.14347\,C_p^{-2.1976}$

In the formulas for $1000C_1$, $1000C_2$, $1000C_3$ and $1000C_4$, the value of C_{WL} is based on the value of i_E in degrees, i.e. $C_{WL} = i_E \cdot L/B$, where i_E is the half angle of entrance of the design waterline. The values of L/B and B/T can be based on the waterline values of L, B and T, although Van Oortmerssen used the displacement length L_D rather than the waterline length, where L_D was defined as half the sum of the length between perpendiculars and the waterline length.

4.4. Numerical Method Derived by Mercier and Savitsky

Mercier and Savitsky [12] carried out a regression analysis of the resistance results obtained by Nordstrom [1] for a small systematic series (9 models), by De Groot [2] for a small systematic series (12 models), by Beys [4] for the series 63 (21 models), by Yeh [5] for the series 64 (27 models), by Lindgren and Williams [6] for the SSPA series (9 models), and by Marwood and Bailey [7] for the NPL series (23 models). The results obtained by Clement and Blount [13] for the series 62 hard-chine hull forms (17 models) were also incorporated in the data base. Formulas were derived for the total resistance-displacement weight ratio R_T/Δ for eleven values of the volumetric Froude number, 1.0, 1.1 ---- 2.0, for a displacement of 100000 lbs. Four parameters were selected for inclusion in the resistance equations as independent variables. These are the displacement-length ratio $\nabla^{1/3}/L$, the beam-loading coefficient $C_\Delta = \nabla/B^3$, the square root of the angle of entrance of the load water line in degrees $\sqrt{2i_E}$, and the ratio of transom area to maximum section area A_T/A_X.

Although the formulas derived by Mercier and Savitsky were originally intended for predicting the resistance of planing craft in the pre-planing, displacement mode, they can also be successfully used for predicting the resistance of displacement hulls, since 101 of the 118 sets of model data used, are those of round-bilge, displacement hulls!

The general form of the resistance equation adopted by Mercier and Savitsky is as follows:

$$\begin{aligned} R_T/\Delta = & \ A_1 + A_2X + A_4U + A_5W + A_6XZ + A_7XU + \\ & + A_8XW + A_9ZU + A_{10}ZW + A_{15}W^2 + \\ & + A_{18}XW^2 + A_{19}ZX^2 + A_{24}UW^2 + A_{27}WU^2 \end{aligned} \tag{18}$$

where

$$X = \nabla^{1/3}/L,\ Z = \nabla/B^3,\ U = \sqrt{2i_E} \text{ and } W = A_T/A_X.$$

The values of the coefficients A_1 to A_{27} are given in Table 4 for the eleven values of the Froude number and a displacement weight of 100000 lbs (444.97375 kN).

Table 4.

Values of the coefficients in Eq. 18 as a function of $F_{n\nabla}$ for Δ = 100000 lbs in sea water at 15°C using the 1947 ATTC friction coefficients with $C_A = 0$.

	$F_{n\nabla} = 1.0$	$F_{n\nabla} = 1.1$	$F_{n\nabla} = 1.2$	$F_{n\nabla} = 1.3$	$F_{n\nabla} = 1.4$	$F_{n\nabla} = 1.5$	$F_{n\nabla} = 1.6$	$F_{n\nabla} = 1.7$	$F_{n\nabla} = 1.8$	$F_{n\nabla} = 1.9$	$F_{n\nabla} = 2.0$
A_1	0.06473	0.10776	0.09483	0.03475	0.03013	0.03163	0.03194	0.04343	0.05036	0.05612	0.05967
A_2	– 0.48680	– 0.88787	– 0.63720	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A_4	– 0.01030	– 0.01634	– 0.01540	– 0.00978	– 0.00664	0.0	0.0	0.0	0.0	0.0	0.0
A_5	– 0.06490	– 0.13444	– 0.13580	– 0.05097	– 0.05540	– 0.10543	– 0.08599	– 0.13289	– 0.15597	– 0.18661	– 0.19758
A_6	0.0	0.0	– 0.16046	– 0.21880	– 0.19359	– 0.20540	– 0.19442	– 0.18062	– 0.17813	– 0.18288	0.20152
A_7	0.10628	0.18186	0.16803	0.10434	0.09612	0.06007	0.06191	0.05487	0.05099	0.04744	0.04645
A_8	0.97310	1.83080	1.55972	0.43510	0.51820	0.58230	0.52049	0.78195	0.92859	1.18569	1.30026
A_9	– 0.00272	– 0.00389	– 0.00309	– 0.00198	– 0.00215	– 0.00372	– 0.00360	– 0.00332	– 0.00308	– 0.00244	– 0.00212
A_{10}	0.01089	0.01467	0.03481	0.04113	0.03901	0.04794	0.04436	0.04187	0.04111	0.04124	0.04343
A_{15}	0.0	0.0	0.0	0.0	0.0	0.08317	0.07366	0.12147	0.14928	0.18090	0.19769
A_{18}	– 1.40962	– 2.46696	– 2.15556	– 0.92663	– 0.95276	– 0.70895	– 0.72057	– 0.95929	– 1.12178	– 1.38644	– 1.55127
A_{19}	0.29136	0.47305	1.02992	1.06392	0.97757	1.19737	1.18119	1.01562	0.93144	0.78414	0.78282
A_{24}	0.02971	0.05877	0.05198	0.02209	0.02413	0.0	0.0	0.0	0.0	0.0	0.0
A_{27}	– 0.00150	– 0.00356	– 0.00303	– 0.00105	– 0.00140	0.0	0.0	0.0	0.0	0.0	0.0

In using equation (18) (and also equations 16 and 17) it is essential to remain within the range of values of the independent variables used in the data base. Gross errors can occur otherwise. For other displacements values, other water temperatures, friction coefficients or C_A-values, equation (18) can be corrected according to the following expression:

$$(R_T/\Delta)_{\mathrm{corr}} = (R_T/\Delta)_{\mathrm{eq.18}} + (C_f^1 - C_{F_{\mathrm{eq.18}}} + C_A)^{1/2} \cdot \cdot \frac{S}{\nabla^{2/3}} \cdot F_{n\nabla}^2 \qquad (19)$$

In equation (19),

$(R_T/\Delta)_{\mathrm{corr}}$ = corrected value of R_T/Δ,

$(R_T/\Delta)_{\mathrm{eq.18}}$ = values of R_T/Δ according to equation (18),

C_F^1 = friction coefficient for alternative displacement, water temperature or friction formulation,

$C_{F_{\mathrm{eq.18}}}$ = friction coefficient according to the 1947 ATTC friction formulation,

C_A = appropriate value of the model-ship correlation factor,

S = wetted surface.

An analysis of the still water value of the wetted surface of the models comprising the data base resulted in the following formula, with an accuracy of ±9% for 95% of the cases comprising the data base.

$$S/\nabla^{2/3} = 2.262 \sqrt{L/\nabla^{1/3}} (1 + 0.046\, B/T + + 0.00287\, (B/T)^2) \qquad (20)$$

4.5. *Numerical method derived by Holtrop and Mennen*

Recently, Holtrop and Mennen [14] published the results of a statistical analysis of the results of resistance tests with 191 models of different types of ships at NSMB, including moderately fast displacement craft. The maximum value of the Froude number of the data base was 0.45, however, restricting the application of the derived formulas to the low speed range of the kind of craft addressed in this paper (as is the case with Van Oortmerssen's formulas). Again, however, Holtrop and Mennen based their formulation for the residual resistance on a theoretical expression for the wave resistance, which should allow some extrapolation to higher Froude Number values.

Contrary to other data discussed in this paper, the expression for the residual resistance derived by Holtrop and Mennen must be used in conjunction with a calculation of the frictional resistance adopting the form factor concept. For high-speed round bilge displacement vessels the Holtrop and Mennen formulas can be used as follows.

The frictional resistance is calculated from:

$$R_F = \tfrac{1}{2} \rho\, S V^2 (C_F(1+k) + C_A) \qquad (21)$$

where S is the still water wetted surface, V the ship speed, C_F the friction coefficient according to the 1957 ITTC formulation, C_A the model-ship correlation factor, and k the form factor accounting for the effect of the three-dimensional hull form on frictional resistance. The value of $1+k$ can be determined from the following formula:

$$1+k = 0.93 + (T/L)^{0.22284} \cdot (B/L_R)^{0.92497} \cdot \cdot (0.95-C_P)^{-0.52145} \cdot (1-C_P + 0.0225\, LCB)^{0.69060} \qquad (22)$$

in which L_R is the length of the run, which can be determined from the following formula:

$$L_R/L = 1-C_P + 0.06\, C_P \cdot LCB/(4C_P-1) \qquad (23)$$

The wetted surface for use in equation (21) can be calculated from the following formula:

$$S = L(2T+B)\sqrt{C_M}(0.4530 + 0.4425\, C_B + - 0.2862\, C_M - 0.003467\, B/T + 0.3696\, C_{WP}) \qquad (24)$$

The formula for the residual resistance is as follows:

$$R_R/\Delta = C \cdot e^{m_1 F_n^d + m_2 \cos(\lambda F_n^{-2})} \qquad (25)$$

in which

$$C = 2223105\, (B/L)^{3.78613} \cdot (T/B)^{1.07961} \cdot \cdot (90-i_E)^{-1.37565}$$

$$m_1 = 0.0140407\, L/T - 1.75254\, \nabla^{1/3}/L + - 4.79323\, B/L - 8.07981\, C_P + + 13.8673\, C_P^2 - 6.984388\, C_P^3$$

$$d = -0.9$$

$$m_2 = -1.69385\, C_P^2\, e^{-0.1/F_n^2}$$

and

$$\lambda = 1.446\, C_P - 0.03\, L/B.$$

The half angle of entrance of the load water line can be determined from:

$$i_E = 125.67\, B/L - 162.25\, C_P^2 + 234.32\, C_P^3 + + 0.155087\, (LCB)^3 \qquad (26)$$

5. Some final remarks

The methods presented in this paper for the estimation of the calm-water resistance of high-speed, round bilge ships is restricted to the estimation of the bare-hull resistance only. No account is given of how to derive the additive resistance of various kinds of appendages such as bilge keels, rudders, propeller shafting, bossings, etc. Also, no systematic attempt has been made to determine the validity and accuracy of the reviewed methods for different types of hull forms as covered by parameters such as $L/\nabla^{1/3}$, L/B, B/T, C_B, etc.

The main aspect dealt with in this paper is the state-of-the-art of estimating the bare-hull resistance of high-speed, round-bilge ships, in the preliminary design stage, with an emphasis on the dependence of the resistance on various hull form parameters to facilitate design decisions.

6. References

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